Spin Coulomb drag in a spin-polarized Luttinger liquid

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The spin Coulomb drag is a distinctive feature of spin-polarized transport. The current of majority spins can induce a current of minority-spin carriers via the transconductivity. The friction is caused by the Coulomb interaction between up and down spin. This interaction reduces the current but does not change the spin polarization. We calculate the conductivities and the transconductivity for a spin-polarized interacting one-dimensional electron gas with nonmagnetic impurities using the Kubo formalism. Due to the Luttinger-liquid properties, the temperature dependence of the transport correlation functions follow power laws of T with nonuniversal exponents.

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I. INTRODUCTION

The possibility of mesoscopic devices based also on spin degrees of freedom (spintronics) in addition to the usual charge transport has raised great interest in spin-polarized transport in metals and semiconductors. Many interesting effects are the result of Coulomb interactions between the carriers in low dimensions. In a one-dimensional (1D) conductor the correlations between electrons lead to exotic properties generically referred to as Luttinger liquid.[1] Characteristic of 1D metallic systems are the charge and spin separation, i.e., the charge and spin contents of the wave functions propagate with different velocities, and the disappearance of the Fermi-liquid quasiparticle pole in the excitation spectrum, which is replaced by incoherent structures. Hence, the Fermi-liquid picture breaks down for interacting electrons in 1D.

Charge carriers moving in one conductor may interact via the Coulomb interaction with carriers in another conductor located nearby. Via momentum conservation the charges in conductor 1 can exert a force on the carriers in the conductor 2 and induce a drag current in the *opposite* direction. This mechanism, now known as the Coulomb drag, was proposed by Pogrebinskii[2] for a semiconductor-insulator-semiconductor layer structure. At very low temperatures the drag effect in a two-dimensional (2D) system is dominated by phonons. The theoretical and experimental developments of the Coulomb-drag effect in a coupled electron system have been reviewed by Rojo.[3]

A Coulomb drag is also present between two parallel quantum wires and has been extensively reviewed in Ref. 4, for both the Fermi-liquid and Luttinger-liquid pictures. While within the Fermi-liquid approach this leads to a drag current proportional to temperature,[5] the Luttinger-liquid picture gives rise to nonuniversal power laws with critical exponents that depend on the interaction strength.[6–9] The Coulomb drag in the presence of a magnetic field has been studied in Ref. 10 and between two spin-incoherent Luttinger liquids in Ref. 11.

The *spin Coulomb drag*,[12] on the other hand, is the decay of the spin current in a metal as the consequence of the Coulomb interactions between different spin populations. In spin-polarized conductors the carriers of opposite spin drift in general with different group velocities but in the same direction. Majority-spin electrons in a ferromagnetic conductor usually move with a larger Fermi velocity than the carriers with minority-spin component. Due to friction between the spin components via the Coulomb interaction the minority-spin electrons are accelerated at the expense of the majority-spin current.[13–18] In the absence of spin-flip mechanisms, such as magnetic impurities, the total magnetization is conserved and only the currents of up- and down-spin carriers changes. While the Coulomb drag between charges in 1D requires two nearby conductors, for the observation of the spin Coulomb drag only one conductor is needed. It is difficult to create devices with parallel electrically isolated quantum wires that are sufficiently long and close enough to yield a measurable drag voltage as required for the Coulomb drag. The spin Coulomb drag may then be more favorable for observation.[17]

The spin Coulomb drag in a quantum wire has been studied by several authors.[19–23] In Ref. 19 the interplay of the spin drag with the spin-charge separation was studied, in Ref. 20 quantum Monte Carlo simulations for the spin-drag conductance of the Hubbard chain were conducted, Sonin[21] investigated the equilibrium spin currents at the edge states of the 2D Rashba medium, i.e., a spin-orbit quantum wire, in Ref. 22 the generation of a spin current by Coulomb drag between two quantum wires via the application of a magnetic field was studied, and the spin-charge separation in a strongly correlated spin-polarized chain was analyzed using path integrals and the bosonization technique in Ref. 23.

We employ the bosonization approach and the Kubo formula to calculate the conductivity in the majority and minority bands and the transconductivity. Bosonization techniques have been applied previously to obtain the conductivity in an unpolarized Luttinger liquid with impurity scattering,[24–27] as well as for a spin-polarized Luttinger liquid.[28–31] In the spin-polarized gas the Fermi velocities for majority and minority spins are assumed to be different. A spin-polarized one-dimensional conductor could, in principle, be fabricated by etching a nanogroove into a locally depleted 2D electron gas in ferromagnetic Mn-doped GaAs.

To obtain a finite transresistivity it is necessary to include impurity scattering and/or inelastic scattering by phonons. We do not include phonons and treat impurity scattering and the spin-flip Coulomb backscattering perturbatively. A perturbative treatment in the disorder is insufficient to include localization and dephasing effects, which are then completely neglected in the present calculation. The conductivity

follows a power law of the temperature with a nonuniversal exponent. Similarly the transconductivity is a linear superposition of two power laws of the temperature with nonuniversal exponents. The power-law behavior is not valid for low energies and low temperatures since the long-wavelength excitations of the electron gas are dampened by the impurity scattering.

The spin-charge separation in 1D is strictly valid only in the absence of spin polarization of the electron gas.[28,29] A spin polarization changes the Luttinger properties substantially. The two spin components have different Fermi velocities and Fermi momenta and the momentum and energy conservation cannot be satisfied simultaneously for all low-energy scattering processes. The transconductivity in the spin-polarized case arises then from a different scattering process than for the unpolarized Luttinger liquid.

The remainder of the paper is organized as follows. In Sec. II we introduce the generalized Kubo equation and the Luttinger liquid with backward scattering. In the Appendix we calculate the conductivity to second order in the Coulomb interaction parameters. Due to momentum conservation this does not lead to an efficient relaxation mechanism. Back scattering across the Fermi surface by *impurities* or inelastic scattering by phonons is necessary to obtain a finite relaxation time as $T\to 0$. In Sec. III the Fermi gas is bosonized and the forward-scattering and non-spin-flip-scattering amplitudes across the Fermi surfaces are diagonalized by means of a canonical transformation. This canonical transformation does no longer decouple charge and spin degrees of freedom as in the absence of spin polarization (no charge and spin separation).[28,29] The conductivities and the transconductivity are calculated using the Luttinger-liquid properties of the interacting gas. Discussions and conclusions follow in Sec. IV.

II. MODEL

In this section we present the Kubo formula for the linear response of a two-component electron gas, state the Hamiltonian for the one-dimensional interacting Fermi gas with backward scattering, and calculate the conductivity and transconductivity to lowest-order perturbation due to the impurity scattering and the interactions.

A. Kubo's equation and memory function

We assume that the dynamical conductivity in linear response to an external electric potential is given by the Kubo formula. The current operator for up- and down-spin electrons is defined as

$$j_s = \sum_k \frac{k}{m}(a_{ks}^\dagger a_{ks} + b_{ks}^\dagger b_{ks}), \tag{1}$$

where $s=\pm 1$ denotes the spin, m is the mass, and a_{ks} and b_{ks} are the annihilation operators for electrons carrying momentum $k>0$ and $k<0$, respectively, with the momentum measured from the center of the Brillouin zone.

In terms of the spin components the conductivity is defined as a 2×2 matrix,

$$\sigma_{s,s'}(z) = -i(e^2/z)\chi_{s,s'}(z) + i(e^2 n_s/mz)\delta_{s,s'}, \tag{2}$$

where e is the electron charge, n_s is the number of carriers with spin component s, and $\chi_{s,s'}(z)=-\langle\langle j_s;j_{s'}\rangle\rangle_z$ is the current-current correlation function.[12,32] The diagonal terms of $\hat{\sigma}$ are the conductivity of the up and down spins, respectively, and the off-diagonal component is the transconductivity. We define $\chi^0_{s,s'}=(n_s/m)\delta_{s,s'}$ as the static limit of $\chi_{s,s'}(z)$.

Following Götze and Wölfle[32] we introduce a memory or relaxation matrix function $M_{s,s'}(z)$,

$$\hat{M}(z) = z\hat{\chi}(z)[\hat{\chi}^0 - \hat{\chi}(z)]^{-1}. \tag{3}$$

The matrix function $\hat{\chi}(z)$ satisfies the inequality $\hat{\chi}(z)\neq\hat{\chi}^0$ for all nonreal z. $\hat{M}(z)$ is an analytic function of z, satisfying $M_{s,s'}(z)=-M_{s,s'}(-z)$ and decreasing asymptotically for large $|z|$ like $1/z$.

$\hat{\chi}(z)$ and $\hat{\sigma}(z)$, expressed in terms of $\hat{M}(z)$ and $\hat{\chi}^0$, are

$$\hat{\chi}(z) = [z\hat{I} + \hat{M}(z)]^{-1}\hat{M}(z)\hat{\chi}^0,$$

$$\hat{\sigma}(z) = ie^2[z\hat{I} + \hat{M}(z)]^{-1}\hat{\chi}^0, \tag{4}$$

which have the advantage of having a resonance structure already build in. In other words, for simple nonmagnetic impurity scattering the memory function is an imaginary constant, which straightforwardly yields a definition for the relaxation rates,[32]

$$\frac{1}{\tau_{s,s'}} = -\lim_{\omega\to 0}\omega\langle\langle j_s;j_{s'}\rangle\rangle''_\omega\frac{m}{n_{s'}}, \tag{5}$$

where $\tau_{s,s}$ refers to the relaxation times of the conductivity and $\tau_{s,\bar{s}}$ to that of the transconductivity. Here $\bar{s}=-s$ and $\langle\langle;\rangle\rangle''_\omega$ refers to the imaginary part of the correlation function.

B. Luttinger liquid with backward scattering

A one-dimensional system of electrons can be described by the Hamiltonian, $H=H_0+H'$,[26,33,34]

$$H_0 = \sum_{k,s} v_{Fs}k(a_{ks}^\dagger a_{ks} - b_{ks}^\dagger b_{ks}),$$

$$H' = \frac{V}{L}\sum_{kpqss'} a_{p+ks}^\dagger a_{ps} b_{q-ks'}^\dagger b_{qs'} + \frac{1}{L}\sum_{kpqss'} a_{p+ks}^\dagger b_{q-ks'}^\dagger a_{ps'} b_{qs}(U_\parallel\delta_{s,s'} + U_\perp\delta_{s,\bar{s}'}), \tag{6}$$

where v_{Fs} is the Fermi velocity for the spin component s and L is the length of the chain. The interaction V corresponds to small momentum transfer while the terms with amplitudes $U_\parallel$ and $U_\perp$ are the large momentum-transfer interactions, without and with spin flip, respectively. They correspond to backward scattering of the electrons across the Fermi "surface" and are important for momentum transfers around $2k_{Fs}$. For the special choice of parameters $U_\parallel=U_\perp=V$ the Hamiltonian reduces to the Hubbard model. The case $U_\parallel=U_\perp=0$ reduces to the Tomonaga-Luttinger model.[35–37] The properties of

model [Eq. (6)] have been extensively studied for the unpolarized situation, i.e, for $v_{F\uparrow}=v_{F\downarrow}$ and $k_{F\uparrow}=k_{F\downarrow}$.

In the Appendix we show that if the total momentum is conserved the relaxation function is given by one scalar function [see Eq. (A2)]. According to Eq. (5) with Coulomb scattering alone a relaxation time cannot be obtained at $T=0$, again as a consequence of the momentum conservation of the Hamiltonian. It is necessary to consider the scattering of electrons off phonons or impurities to break the translational invariance of the electrons. In this paper we consider the effect of impurities.

C. Impurity scattering

The scattering of the electrons off impurities is defined by the Hamiltonian

$$H_{imp}=\frac{\lambda}{L}\sum_{jkk's} e^{i(k'-k)R_j}[a_{ks}^{\dagger}a_{k's}+b_{ks}^{\dagger}b_{k's}+a_{ks}^{\dagger}b_{k's}+b_{ks}^{\dagger}a_{k's}], \tag{7}$$

where λ is the coupling strength, which is assumed to be weak, and R_j denote the positions of the scattering centers. We assume that the impurities are distributed at random and that their concentration is low so that any interference among the scattering centers can be disregarded.[32] This eliminates any possibility for a localization of states due to disorder. Equation (7) has two types of terms; the first two terms involve only forward scattering, i.e., small momentum transfer while the latter two terms consist of backward scattering, i.e., across the Fermi surface with momentum transfer $2k_{Fs}$.[24] Recall that for a spin-polarized electron gas k_{Fs} depends on the spin component. Since H_{imp} does not commute with the total momentum of the carriers, the expectation value of the momentum is not a conserved quantity. This process gives rise to a finite resistivity.

D. Conductivity in second-order perturbation

In the presence of impurities the force operator acting on the current of spin component s is

$$\begin{aligned}A_s&=[j_s,H_{imp}]\\&=\frac{\lambda}{mL}\sum_{jkk'}(k-k')e^{i(k'-k)R_j}\\&\quad\times(a_{ks}^{\dagger}a_{k's}+b_{ks}^{\dagger}b_{k's}+a_{ks}^{\dagger}b_{k's}+b_{ks}^{\dagger}a_{k's})\end{aligned} \tag{8}$$

and to second order in λ the imaginary part of the equal spin current-current correlation function is

$$\begin{aligned}\omega^2\langle\langle j_s;j_s\rangle\rangle''_{\omega}&=-\langle\langle A_s;A_s\rangle\rangle''_{\omega}\\&=-\left(\frac{\lambda}{mL}\right)^2 n_i\sum_{kk'}(k-k')^2[\langle\langle a_{ks}^{\dagger}a_{k's};a_{k's}^{\dagger}a_{ks}\rangle\rangle''_{\omega}\\&\quad+\langle\langle b_{ks}^{\dagger}b_{k's};b_{k's}^{\dagger}b_{ks}\rangle\rangle''_{\omega}+\langle\langle a_{ks}^{\dagger}b_{k's};b_{k's}^{\dagger}a_{ks}\rangle\rangle''_{\omega}\\&\quad+\langle\langle b_{ks}^{\dagger}a_{k's};a_{k's}^{\dagger}b_{ks}\rangle\rangle''_{\omega}],\end{aligned} \tag{9}$$

where n_i/L is the impurity density. Here we assumed that there is no interference in the scattering of electrons off the impurities. The first two terms correspond to forward scattering and the momentum transfer is small. The contribution arising from these terms is $-(\lambda^2 n_i/2\pi)(\omega/v_{Fs})^3$, i.e., $M_{s,s}(\omega)$ is proportional to ω^2. Hence, forward scattering of electrons off the impurities does not contribute to a finite relaxation time.

The last two terms in Eq. (9) correspond to backward scattering of electrons off the impurities and involves a momentum transfer of $2k_{Fs}$. Hence, the factor $(k-k')^2$ is not small and the contribution to $M_{s,s}(\omega)$ is two powers of ω less than the forward-scattering terms. Evaluated for the noninteracting system these contributions yield $-2\lambda^2 n_i\omega/\pi$ so that the relaxation function is now constant as $\omega\to 0$.[24]

The relaxation rate for up-spin and down-spin electrons is in general different and given by $1/\tau_{s,s}=2\lambda^2 n_i m/\pi n_s$.[32] Note that the transconductivity is zero because H_0+H_{imp} conserves the spin. It is necessary to consider the interplay between impurity scattering and the Coulomb interaction to generate a transconductivity.

E. Transconductivity

From the above analysis it is clear that each forward scattering yields a factor of ω in view of the small momentum transfer. It is therefore more effective to consider the $U_{\perp}$ interaction rather than the V interaction. Note that the $U_{\parallel}$ term in the Hamiltonian only involves one spin component and cannot give rise to a transconductivity.

For the Hamiltonian $H=H_0+H_{imp}+H_{U_{\perp}}$ the force operator acting on the current of spin component s is

$$\begin{aligned}A_s&=[j_s,H]\\&=\frac{\lambda}{mL}\sum_{jkk'}(k-k')e^{i(k'-k)R_j}(a_{ks}^{\dagger}b_{k's}+b_{ks}^{\dagger}a_{k's})\\&\quad+\frac{U_{\perp}}{mL}\sum_{kk'qs'}ss'(k+q-k')a_{k+qs'}^{\dagger}b_{k'-q\bar{s}'}^{\dagger}a_{k\bar{s}'}b_{k's'},\end{aligned} \tag{10}$$

where we neglected the impurity forward scattering since it gives rise to an additional factor ω. The lowest-order contributing to the off-diagonal conductivity with a finite relaxation time is $U_{\perp}\lambda^2$.

We factorize the $U_{\perp}$ term in Eq. (10) in the usual way,

$$\begin{aligned}&a_{k+qs'}^{\dagger}b_{k'-q\bar{s}'}^{\dagger}a_{k\bar{s}'}b_{k's'}\\&\quad\to\langle a_{k+qs'}^{\dagger}b_{k's'}\rangle b_{k'-q\bar{s}'}^{\dagger}a_{k\bar{s}'}+\langle b_{k'-q\bar{s}'}^{\dagger}a_{k\bar{s}'}\rangle a_{k+qs'}^{\dagger}b_{k's'}.\end{aligned} \tag{11}$$

Each of the factors conserves the spin and the expectation values are evaluated using H_{imp}. The spin-up and spin-down current correlation function is now given by

$$\omega^2\langle\langle j_s;j_{\bar{s}}\rangle\rangle''_\omega = -\frac{\lambda U_\perp}{m^2L^2}\sum_{jkk'q}(k-k')^2[e^{i(k'-k)R_j}\langle a^\dagger_{k+q\bar{s}}b_{k'+q\bar{s}}\rangle\langle\langle a^\dagger_{ks}b_{k's};b^\dagger_{k's}a_{ks}\rangle\rangle''_\omega + e^{-i(k'-k)R_j}\langle b^\dagger_{k-q\bar{s}}a_{k'-q\bar{s}}\rangle\langle\langle b^\dagger_{ks}a_{k's};a^\dagger_{k's}b_{ks}\rangle\rangle''_\omega$$
$$+ e^{-i(k'-k)R_j}\langle b^\dagger_{k'-qs}a_{k-qs}\rangle\langle\langle a^\dagger_{k\bar{s}}b_{k'\bar{s}};b^\dagger_{k'\bar{s}}a_{k\bar{s}}\rangle\rangle''_\omega + e^{i(k'-k)R_j}\langle a^\dagger_{k'+qs}b_{k+qs}\rangle\langle\langle b^\dagger_{k\bar{s}}a_{k'\bar{s}};a^\dagger_{k'\bar{s}}b_{k\bar{s}}\rangle\rangle''_\omega]. \quad (12)$$

The correlation functions are evaluated for the noninteracting system yielding

$$\omega^2\langle\langle j_s;j_{\bar{s}}\rangle\rangle''_\omega = -\frac{\lambda U_\perp}{m^2L^2}\pi\sum_{jkk'q}(k-k')^2\{e^{i(k'-k)R_j}\langle a^\dagger_{k+q\bar{s}}b_{k'+q\bar{s}}\rangle\delta(\omega+\epsilon_{ks}-\epsilon_{k's})[f(\epsilon_{ks})-f(\epsilon_{k's})]$$
$$+ e^{-i(k'-k)R_j}\langle b^\dagger_{k-q\bar{s}}a_{k'-q\bar{s}}\rangle\delta(\omega+\epsilon_{ks}-\epsilon_{k's})[f(\epsilon_{ks})-f(\epsilon_{k's})] + e^{-i(k'-k)R_j}\langle b^\dagger_{k'-qs}a_{k-qs}\rangle\delta(\omega+\epsilon_{k\bar{s}}-\epsilon_{k'\bar{s}})[f(\epsilon_{k\bar{s}})-f(\epsilon_{k'\bar{s}})]$$
$$+ e^{i(k'-k)R_j}\langle a^\dagger_{k'+qs}b_{k+qs}\rangle\delta(\omega+\epsilon_{k\bar{s}}-\epsilon_{k'\bar{s}})[f(\epsilon_{k\bar{s}})-f(\epsilon_{k'\bar{s}})]\}. \quad (13)$$

Here we denoted $\epsilon_{ks}=v_{Fs}|k|$ for both, forward and backward, movers. The expectation values in Eq. (13) are evaluated using $H=H_0+H_{imp}$ and the propagator $\langle\langle b_{qs};a^\dagger_{qs}\rangle\rangle$, e.g.,

$$\langle a^\dagger_{qs}b_{ps}\rangle = \frac{\lambda}{L}\sum_j e^{-i(q+p)R_j}\frac{f(\epsilon_{qs})-f(\epsilon_{ps})}{\epsilon_{qs}-\epsilon_{ps}}, \quad (14)$$

and the other ones yield a similar result. When inserted into Eq. (13) the sum over j just yields the number of impurities, n_i, where we again assumed that there is no interference in the scattering between impurities. In all terms the momentum transfer in the scattering is either $\pm 2k_{F\uparrow}$ or $\pm 2k_{F\downarrow}$. The opposite spin current-current correlation function then reduces to

$$\omega^2\langle\langle j_s;j_{\bar{s}}\rangle\rangle''_\omega = -\left(\frac{\lambda}{mL}\right)^2\frac{U_\perp}{L}2\pi n_i\sum_{kk'q}(2k_{Fs})^2$$
$$\times\frac{f(\epsilon_{k+q\bar{s}})-f(\epsilon_{k'+q\bar{s}})}{\epsilon_{k+q\bar{s}}-\epsilon_{k'+q\bar{s}}}\delta(\omega+\epsilon_{ks}-\epsilon_{k's})$$
$$\times[f(\epsilon_{ks})-f(\epsilon_{k's})]+(s\leftrightarrow\bar{s}). \quad (15)$$

Converting the sums over momenta into integrals we obtain

$$\omega\langle\langle j_s;j_{\bar{s}}\rangle\rangle''_\omega = -\frac{\lambda^2U_\perp n_i m}{\pi^2}\left(\frac{1}{k_{F\bar{s}}}+\frac{1}{k_{Fs}}\right), \quad (16)$$

i.e., the transconductivity is proportional to the impurity concentration, to the backscattering amplitude $U_\perp$, the square of the impurity scattering and inversely proportional to the group velocities. So far we calculated the lowest-order contributions and have not yet taken into account the Luttinger-liquid properties of the electron gas.

III. LUTTINGER-LIQUID PROPERTIES

A. Bosonization

The logarithmic correction appearing to all order in perturbation in interacting one-dimensional electron systems are best taken into account by bosonizing the V and $U_\parallel$ terms in Hamiltonian (6). The low-energy excitations of the electron gas can be described by charge- and spin-density wave operators defined as[25,26,34]

$$\rho_{1s}(k)=\sum_p a^\dagger_{p+ks}a_{ps},\quad \rho_{2s}(k)=\sum_p b^\dagger_{p+ks}b_{ps}, \quad (17)$$

which satisfy the following boson commutation relations:

$$[\rho_{1s}(-k),\rho_{1s'}(k')]=[\rho_{2s}(k),\rho_{2s'}(-k')]=\frac{kL}{2\pi}\delta_{kk'}\delta_{ss'} \quad (18)$$

while all other commutators vanish.

The fermion field operators $\psi_{1s}(x)=L^{-1/2}\Sigma_k e^{ikx}a_{ks}$ and $\psi_{2s}(x)=L^{-1/2}\Sigma_k e^{ikx}b_{ks}$ can be expressed in terms of the boson operators[25,26,34]

$$\psi_{js}(x)=(2\pi\alpha)^{-1/2}\exp\{(-1)^{j+1}[ik_{Fs}x+\varphi_{js}(x)]\},$$
$$\varphi_{js}(x)=(2\pi/L)\sum_k k^{-1}e^{-\alpha|k|/2-ikx}\rho_{js}(k), \quad (19)$$

where α is a cutoff for the electronic excitations, which should be on the order of the inverse of the bandwidth. The relation (19), however, is only rigorously correct in the limit $\alpha\to 0$ and if a charge operator is included in the exponent.

With the definitions of the boson operators $H=H_0+H_V+H_{U_\parallel}$ can be rewritten as

$$H=\frac{2\pi}{L}\sum_{ks}\frac{k_{Fs}}{m}[\rho_{1s}(k)\rho_{1s}(-k)+\rho_{2s}(-k)\rho_{2s}(k)]$$
$$+\frac{V}{L}\sum_{kss'}\rho_{1s}(k)\rho_{2s'}(-k)-\frac{U_\parallel}{L}\sum_{ks}\rho_{1s}(k)\rho_{2s}(-k). \quad (20)$$

This expression is bilinear in boson operators and can be diagonalized by means of a canonical transformation.[26] For the unpolarized electron gas, i.e., $k_{F\uparrow}=k_{F\downarrow}$, H can be decomposed into $H_\rho+H_\sigma$, where $[H_\rho,H_\sigma]=0$, and H_ρ and H_σ represent charge- and spin-density waves, respectively. This leads to the well-known charge-spin separation in one dimension.[26,33,34]

If $k_{F\uparrow}\neq k_{F\downarrow}$ there is no charge-spin separation and the four operators $\rho_{1\uparrow}(k)$, $\rho_{1\downarrow}(k)$, $\rho_{2\uparrow}(k)$, and $\rho_{2\downarrow}(k)$ are all coupled. It

is then necessary to diagonalize a 4×4 matrix (rather than two 2×2 matrices). A partial diagonalization, i.e., for $U_\parallel=0$, has been studied in Ref. 28 and the general case in Ref. 29. The 4×4 matrix can be constructed from the equations of motion of the above-mentioned operators, e.g., $id\rho_j(k)/dt=[\rho_j(k),H]$, where $j=1,\ldots,4$ labels the operators in the order they are listed above. Fourier transforming with respect to time and denoting $x=-2\pi\omega/k$, the 4×4 matrix related to the eigenvalue problem is

$$\begin{pmatrix} x+2\pi v_{F\uparrow} & 0 & V-U_\parallel & V \\ 0 & x+2\pi v_{F\downarrow} & V & V-U_\parallel \\ -V+U_\parallel & -V & x-2\pi v_{F\uparrow} & 0 \\ -V & -V+U_\parallel & 0 & x-2\pi v_{F\downarrow} \end{pmatrix} \tag{21}$$

and its determinant equated to zero yields the velocities for the four dispersions, $\omega_j=v_jk$, $j=1,\ldots,4$, where $v_3=-v_1$ and $v_4=-v_2$. Note that the matrix is not Hermitian. The eigenvectors associated with these dispersions can be written as

$$\tilde{\rho}_j(k)=\sum_{l=1}^{4}\alpha_{jl}\rho_l(k) \tag{22}$$

and conversely we may express ρ_j in terms of $\tilde{\rho}_j$ by inverting the matrix

$$\rho_l(k)=\sum_{j=1}^{4}\beta_{lj}\tilde{\rho}_j(k). \tag{23}$$

The eigenvalues and eigenvectors are obtained numerically. The Hamiltonian is diagonal in the new basis of states

$$H=\frac{2\pi}{L}\sum_{j=1}^{2}v_j[\tilde{\rho}_j(k)\tilde{\rho}_j(-k)+\tilde{\rho}_{j+2}(-k)\tilde{\rho}_{j+2}(k)] \tag{24}$$

and the conductivity and transconductivity are to be computed in this new basis set.

The group velocities of the diagonalized Hamiltonian are shown in Fig. 1 as a function of V for three values of $U_\parallel$. The Coulomb interactions reduce the velocities since the particles drag electron-hole excitations. Note that the velocity in the minority band is reduced much more than the one in the majority band. The key parameter in the perturbation is the scattering amplitude V or $U_\parallel$ times the density of states (inversely proportional to the Fermi velocity). Since the Fermi velocity for the minority band is much smaller than that of the majority band, the effect of the perturbation is stronger for the minority band.

B. Conductivity

The starting point to obtain the up-spin and down-spin conductivity to all orders in V and $U_\parallel$ are the third and fourth correlation functions in expression (9). The momentum transfer for both terms is $\pm2k_{Fs}$, i.e., from one Fermi point to the other. In the presence of interactions we have to consider the more general form

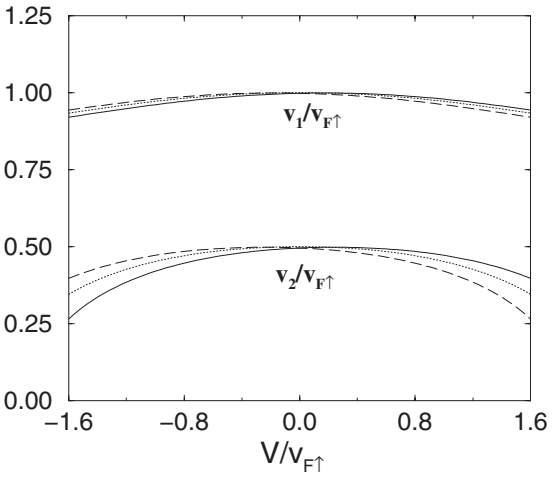

FIG. 1. Group velocities as a function of the forward-scattering amplitude V for $U_\parallel=0.4v_{F\uparrow}$ (solid curves), $U_\parallel=0.0$ (dotted curves), and $U_\parallel=-0.4v_{F\uparrow}$ (long-dashed curves). Here the bare Fermi velocities are $v_{F\uparrow}=1.0$ and $v_{F\downarrow}=0.5$, respectively.

$$\sum_{kk'q}[\langle\langle a^\dagger_{k+qs}b_{k's};b^\dagger_{k'-qs}a_{ks}\rangle\rangle''_\omega+\langle\langle b^\dagger_{k'-qs}a_{ks};a^\dagger_{k+qs}b_{k's}\rangle\rangle''_\omega]. \tag{25}$$

The creation and annihilation operators can be expressed by field operators

$$a^\dagger_{ps}=L^{-1/2}\int dxe^{-ipx}\psi^\dagger_{1s}(x),$$

$$b^\dagger_{ps}=L^{-1/2}\int dxe^{-ipx}\psi^\dagger_{2s}(x), \tag{26}$$

etc. The sequence of operators for the first correlation function is then

$$\sum_{kk'q}a^\dagger_{k+qs}(t)b_{k's}(t)b^\dagger_{k'-qs}(0)a_{ks}(0)$$
$$=L\int dx\psi^\dagger_{1s}(x,t)\psi_{2s}(x,t)\psi^\dagger_{2s}(x,0)\psi_{1s}(x,0). \tag{27}$$

In bosonized form the operator takes the form

$$\frac{L}{(2\pi\alpha)^2}\int dx\exp[-\varphi_{1s}(x,t)]\exp[-\varphi_{2s}(x,t)]$$
$$\times\exp[\varphi_{2s}(x,0)]\exp[\varphi_{1s}(x,0)] \tag{28}$$

and similarly the Hermitian conjugated operator.

We now apply the same canonical transformation that diagonalized $H_0+H_V+H_{U_\parallel}$ to the operator [Eq. (28)], yielding

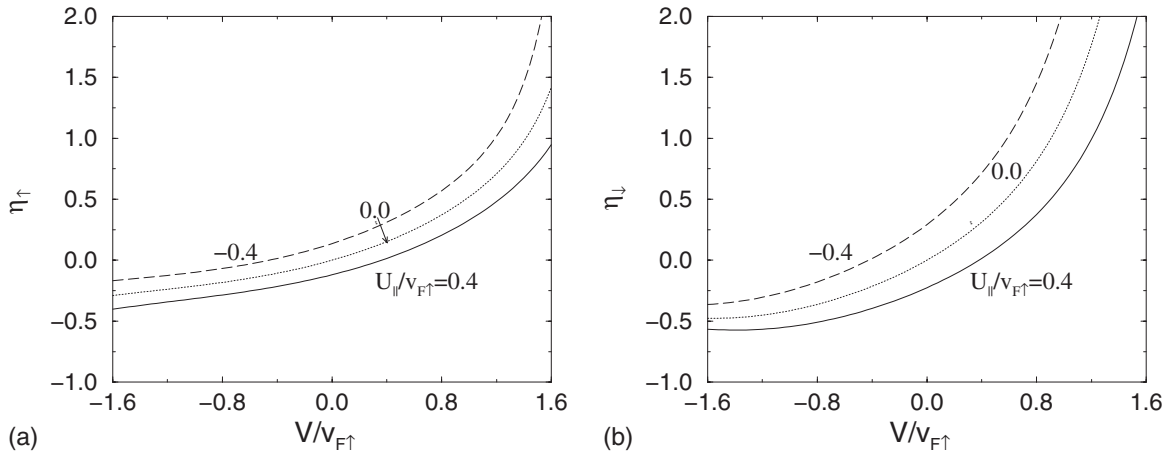

FIG. 2. Critical exponents η_s for the conductivity of (a) up-spin carriers and (b) down-spin carriers as a function of the forward-scattering amplitude V for $U_\parallel=0.4v_{F\uparrow}$ (solid curves), $U_\parallel=0.0$ (dotted curves), and $U_\parallel=-0.4v_{F\uparrow}$ (long-dashed curves). Here the bare Fermi velocities are $v_{F\uparrow}=1.0$ and $v_{F\downarrow}=0.5$, respectively.

$$\frac{L}{(2\pi\alpha)^2}\int dx \times\exp\left[-\frac{2\pi}{L}\sum_k k^{-1}e^{-\alpha|k|/2-ikx}\sum_{j=1}^{4}\beta_{1j}\tilde{\rho}_j(k)e^{-iv_jkt}\right] \times\exp\left[-\frac{2\pi}{L}\sum_k k^{-1}e^{-\alpha|k|/2-ikx}\sum_{j=1}^{4}\beta_{3j}\tilde{\rho}_j(k)e^{-iv_jkt}\right] \times\exp\left[\frac{2\pi}{L}\sum_k k^{-1}e^{-\alpha|k|/2-ikx}\sum_{j=1}^{4}\beta_{3j}\tilde{\rho}_j(k)\right] \times\exp\left[\frac{2\pi}{L}\sum_k k^{-1}e^{-\alpha|k|/2-ikx}\sum_{j=1}^{4}\beta_{1j}\tilde{\rho}_j(k)\right] \tag{29}$$

for up-spin carriers, i.e., $s=\uparrow$. The same expression holds for down-spin carriers but with $\beta_{1j}\to\beta_{2j}$ and $\beta_{3j}\to\beta_{4j}$. The operators can be brought into one common exponential by using $e^Ae^B=e^{A+B+[A,B]/2}$ since the commutator $[A,B]$ is a c number.[25,34]

The correlation function is given by the thermal expectation value of the operator. The expectation value is evaluated by using $\langle e^{\tilde{L}}\rangle=\exp(\langle\tilde{L}^2\rangle/2)$, where $\tilde{L}$ is a linear combination of boson operators. The calculation is tedious but straightforward; for up spins, including the commutator $[A,B]$, we obtain[25,34]

$$\frac{1}{2}\langle\tilde{L}^2\rangle=\sum_{j=1}^{4}(\beta_{1j}+\beta_{3j})^2\frac{2\pi}{L}\sum_k k^{-1}e^{-\alpha|k|} \times\frac{\cos[v_jk(t-r_ji\beta/2)]-\cosh(v_jk/2T)}{\sinh(v_jk/2T)}, \tag{30}$$

where $r_j=1$ for $j=1,2$, $r_j=-1$ for $j=3,4$ and β is the inverse of the temperature. The sum over k can be converted into an integral, yielding[34]

$$\sum_{j=1}^{4}(\beta_{1j}+\beta_{3j})^2\ln\left\{\frac{\alpha}{\alpha-ir_jv_jt}\frac{\pi t/\beta}{\sinh(\pi t/\beta)}\right\}. \tag{31}$$

Equation (31) is the argument of the exponential. Fourier transforming and collecting prefactors we obtain the imaginary part of the correlation function,

$$-\frac{\lambda^2 4k_{F\uparrow}^2}{m^2}\frac{2n_i}{(2\pi\alpha)^2}\prod_{j=1}^{4}\left(\frac{2\pi\alpha T}{|v_j|}\right)^{(\beta_{1j}+\beta_{3j})^2} \times\sinh\left(\frac{\omega}{2T}\right)\left|\Gamma\left(q_\uparrow^2+i\frac{\omega}{2\pi T}\right)\right|^2\frac{1}{\Gamma(2q_\uparrow^2)2\pi T}, \tag{32}$$

where $2q_\uparrow^2=\Sigma_{j=1}^4(\beta_{1j}+\beta_{3j})^2$ and Γ is the gamma function. For $q_\uparrow^2\sim 1$ the above expression can be reduced approximately to[34]

$$\omega\langle\langle j_\uparrow;j_\uparrow\rangle\rangle''_\omega=-\lambda^2n_i\frac{2}{\pi}\frac{1}{\Gamma(2q_\uparrow^2)}\prod_{j=1}^{4}\left(\frac{v_{F\uparrow}}{|v_j|}\right)^{(\beta_{1j}+\beta_{3j})^2} \times\left(\frac{\omega^2+(\pi T)^2}{D_\uparrow^2}\right)^{q_\uparrow^2-1}, \tag{33}$$

where $D_\uparrow=v_{F\uparrow}/(2\pi\alpha)$ is the energy cutoff for the excitations. The expression for the current-current correlation function for down spins is similar and is obtained by changing $v_{F\uparrow}\to v_{F\downarrow}$, $D_\uparrow\to D_\downarrow$, $\beta_{1j}\to\beta_{2j}$, $\beta_{3j}\to\beta_{4j}$, and $q_\uparrow\to q_\downarrow$.

For the noninteracting system, i.e., $q_s^2\to 1$, expression (33) reduces to that of Sec. II D. For the Luttinger liquid the relaxation rate follows a power law of T with exponent $\eta_s=2(q_s^2-1)$. Note that for the spin-polarized system the exponent is different for the conductivity of up spins than for the down spins.[28,29] The exponents are shown in Fig. 2 as a function of V for three values of $U_\parallel$. Note that the exponent for the minority spins varies much more dramatically than the one of the majority spins for the same reason as $v_{F\downarrow}$ has a stronger reduction than $v_{F\uparrow}$. In general, for repulsive V the exponents are positive while they are negative for attractive V. The conductivity can then become large if $V<0$.

C. Transconductivity

The calculation of the transconductivity is similar to that of the conductivity. The main difference arises from the factorization in Eq. (11) because the transconductivity is now a sum over products of an expectation value and a correlation function, Eq. (12). The fermion operators in the correlation function are bosonized in analogy to the procedure for the conductivity in Sec. III B.

To evaluate the expectation values, e.g., $\langle a_{qs}^{\dagger} b_{ps}\rangle$, we again consider a propagator of the form

$$\langle\langle b_{ps};a_{qs}^{\dagger}\rangle\rangle_z \simeq \frac{\lambda}{L}\sum_{jk}\frac{e^{2ik_{Fs}R_j}}{z-\epsilon_{ps}}\langle\langle a_{ks};a_{qs}^{\dagger}\rangle\rangle_z \simeq \frac{\lambda}{L}\sum_{j}\frac{e^{2ik_{Fs}R_j}}{z-\epsilon_{ps}}\langle\langle a_{qs};a_{qs}^{\dagger}\rangle\rangle_z, \tag{34}$$

which is rewritten in terms of field operators and then bosonized,[34]

$$\sum_j \frac{e^{2ik_{Fs}R_j}}{z_n-\epsilon_{ps}}\frac{(-i)\lambda}{2\pi\alpha L^2}\int dx\int dy\int_0^{-i\beta} dt e^{iz_n t-iq(x-y)},$$

$$\langle\exp[\varphi_{1s}(x,t)]\exp[-\varphi_{1s}(y,0)]\rangle, \tag{35}$$

where z_n is a fermion Matsubara pole. We now consider $s=\uparrow$ and apply the canonical transformation that diagonalizes $H_0+H_V+H_{U_\parallel}$. The second line of Eq. (35) is then

$$\left\langle \exp\left[\sum_l \beta_{1l}\varphi_l(x,t)\right]\exp\left[-\sum_l \beta_{1l}\varphi_l(y,0)\right]\right\rangle = \prod_{l=1}^{4}\left(\frac{ir_l\pi\alpha T/v_l}{\sinh[\pi T(x-y-v_l t)]}\right)^{\beta_{1l}^2} \tag{36}$$

and the propagator can be reduced to

$$\langle\langle b_{p\uparrow};a_{q\uparrow}^{\dagger}\rangle\rangle_z \simeq \frac{\lambda}{L}\sum_j \frac{e^{2ik_{F\uparrow}R_j}}{(z-\epsilon_{p\uparrow})(z-\epsilon_{q\uparrow})} \times\left[\prod_{l=1}^{4}\left(\frac{v_{F\uparrow}}{|v_l|}\right)^{\beta_{1l}^2}\right]\left[\frac{\omega^2+(\pi T)^2}{D_\uparrow^2}\right]^{(\tilde{q}_1^2-1)/2}, \tag{37}$$

where $\tilde{q}_j^2=\Sigma_{l=1}^4\beta_{jl}^2$ and similar for down spins. The expectation value is then given by [Eq. (14)] times the factor,

$$\left[\prod_{l=1}^{4}\left(\frac{v_{F\uparrow}}{|v_l|}\right)^{\beta_{1l}^2}\right]\left(\frac{\pi T}{D_\uparrow}\right)^{(\tilde{q}_1^2-1)}, \tag{38}$$

which is the reduction in the discontinuity of the Fermi function at the Fermi level. For $T\to 0$ the discontinuity is suppressed with a nonuniversal power law (marginal Fermi liquid) as expected for a Luttinger liquid. The correlation functions in Eq. (12) are evaluated in analogy to Sec. III B.

The $\omega\to 0$ transconductivity consists of four terms,

$$\omega\langle\langle j_\uparrow;j_\downarrow\rangle\rangle_\omega'' = -\frac{\lambda^2U_\perp n_i}{2\pi^2v_{F\downarrow}\Gamma(2q_\uparrow^2)}\left[\prod_{l=1}^{4}\left(\frac{v_{F\uparrow}}{|v_l|}\right)^{(\beta_{1l}+\beta_{3l})^2}\right]\left(\frac{\pi T}{D_\uparrow}\right)^{2(q_\uparrow^2-1)}\left\{\left[\prod_{l=1}^{4}\left(\frac{v_{F\downarrow}}{|v_l|}\right)^{\beta_{2l}^2}\right]\left(\frac{\pi T}{D_\downarrow}\right)^{\tilde{q}_2^2-1}+\left[\prod_{l=1}^{4}\left(\frac{v_{F\downarrow}}{|v_l|}\right)^{\beta_{4l}^2}\right]\left(\frac{\pi T}{D_\downarrow}\right)^{\tilde{q}_4^2-1}\right\}$$
$$-\frac{\lambda^2U_\perp n_i}{2\pi^2v_{F\uparrow}\Gamma(2q_\downarrow^2)}\left[\prod_{l=1}^{4}\left(\frac{v_{F\downarrow}}{|v_l|}\right)^{(\beta_{2l}+\beta_{4l})^2}\right]\left(\frac{\pi T}{D_\downarrow}\right)^{2(q_\downarrow^2-1)}\left\{\left[\prod_{l=1}^{4}\left(\frac{v_{F\uparrow}}{|v_l|}\right)^{\beta_{3l}^2}\right]\left(\frac{\pi T}{D_\uparrow}\right)^{\tilde{q}_3^2-1}+\left[\prod_{l=1}^{4}\left(\frac{v_{F\uparrow}}{|v_l|}\right)^{\beta_{1l}^2}\right]\left(\frac{\pi T}{D_\uparrow}\right)^{\tilde{q}_1^2-1}\right\}. \tag{39}$$

Due to the V and $U_\parallel$ interactions, the relaxation rate is temperature dependent and given by the sum of four power laws of T with exponents $\eta_1=2(q_\uparrow^2-1)+(\tilde{q}_2^2-1)$, $\eta_2=2(q_\uparrow^2-1)+(\tilde{q}_4^2-1)$, $\eta_3=2(q_\downarrow^2-1)+(\tilde{q}_3^2-1)$, and $\eta_4=2(q_\downarrow^2-1)+(\tilde{q}_1^2-1)$, respectively. It has been numerically verified that $\eta_1=\eta_2$ and $\eta_3=\eta_4$. The exponents are shown in Fig. 3 as a function of V for three values of $U_\parallel$. The exponents are positive for sufficiently large forward-scattering amplitude but may be negative (divergent correlation function as $T\to 0$) for small or negative values of V. The small transconductivity regime (large V) is expected to lead to a strong spin Coulomb-drag effect at low temperatures.

IV. DISCUSSIONS AND CONCLUSIONS

We studied the spin Coulomb drag between the majority- and minority-spin components in a spin-polarized quantum wire. The momentum relaxation, i.e., the increase in the momentum of the minority carriers at the expense of the majority electrons without change in the magnetization, is only effective with impurity scattering. The back scattering of electrons off nonmagnetic impurities, i.e., across the Fermi surface, is essential to produce a finite relaxation rate. We assumed that the impurities are placed at random along the chain and are sufficiently dilute so that interference in the scattering between impurities can be neglected. This excludes all possibility for a localization of states due to disorder. Another necessary ingredient for the spin Coulomb drag to take place is the backscattering spin-flip Coulomb amplitude $U_\perp$ that connects the up-spin and down-spin Fermi surfaces. To lowest order in perturbation the transconductance is then proportional to $\lambda^2U_\perp$ (see Sec. II E).

The temperature dependence of the conductivity and transconductivity is introduced by the Luttinger-liquid prop-

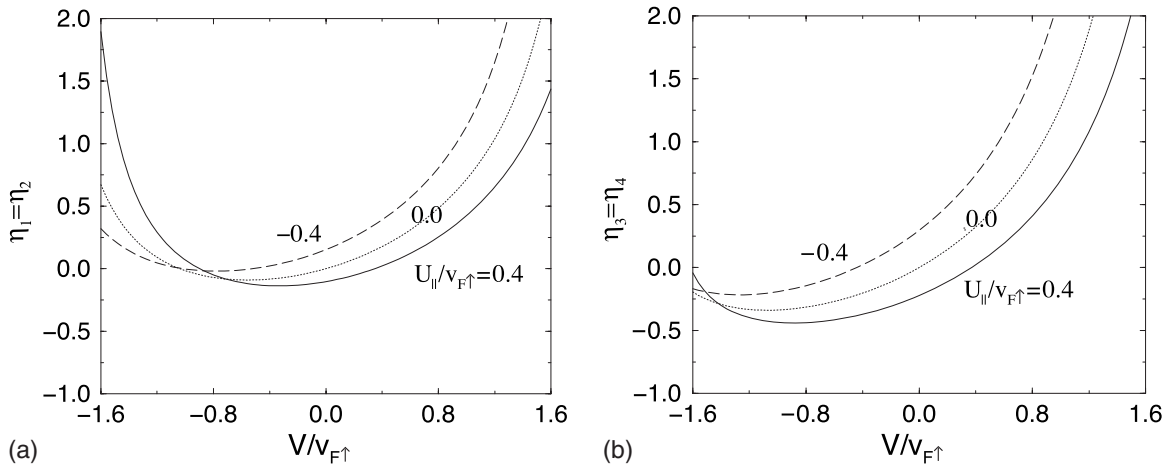

FIG. 3. Critical exponents for the transconductivity (a) $\eta_1=\eta_2$ and (b) $\eta_3=\eta_4$ as a function of the forward-scattering amplitude V for $U_\parallel=0.4v_{F\uparrow}$ (solid curves), $U_\parallel=0.0$ (dotted curves), and $U_\parallel=-0.4v_{F\uparrow}$ (long-dashed curves). Here the bare Fermi velocities are $v_{F\uparrow}=1.0$ and $v_{F\downarrow}=0.5$, respectively.

erties of the 1D electron gas. The kinetic energy of the spin-polarized gas, the forward-scattering amplitude, and the non-spin-flip backward-scattering amplitude and have been diagonalized by means of a canonical transformation. The new collective excitations are a combination of charge- and spin-density waves, have forward and backward moving branches, and are characterized by two group velocities. In contrast to the unpolarized Luttinger liquid, where the charge- and spin-density wave decouple, there is no charge-spin separation in the polarized electron gas.[28,29] The group velocities are reduced as a consequence of the interactions since the particles have to drag the collective excitations. The renormalization of the smaller group velocity, emerging from the minority-spin Fermi velocity in the absence of interactions, is much stronger than that of the majority branch. The reason for this difference is that the perturbation parameters V/v_{Fs} and $U_\parallel/v_{Fs}$ are larger for the minority band. The canonical transformation leads to power-law dependences of the correlation functions.

For repulsive V-interaction strengths the critical exponents of the conductivity are in general positive, indicating that the relaxation times increase when T is reduced, favoring this way the conduction. For attractive interaction ($V<0$), on the other hand, the exponents become negative. In this case the relaxation rate increases, reducing the conduction. Similarly, for the relaxation rate associated with the transconductivity the exponents are positive for large $|V|/v_{F\uparrow}$ but can become negative for weak forward scattering.

The spin Coulomb drag is proportional to the transresistivity.[12] To obtain the transresistivity the conductivity matrix has to be inverted. The leading exponent of the temperature dependence of the transresistivity, η, is shown in Fig. 4 for the same parameters as in the previous figures. There are several power laws contributing to $\rho_{\uparrow\downarrow}$. The irregularities observed in the curves in Fig. 4 are due to crossovers between these exponents. A repulsive forward-scattering amplitude is most favorable for the spin Coulomb drag. Also an attractive $U_\parallel$ enhances the exponent η.

The power laws arising from the Luttinger-liquid properties have the underlying assumption of the long-time approximation. However, the electron states are not infinitely lived due to the scattering of electrons off the impurities. Hence, the power laws have to be cutoff below a certain frequency or temperature, being roughly constant for smaller energies. In other words, the conductivities or transconductivity cannot become arbitrarily large if the corresponding exponent is negative.

The spin-flip backward-scattering process $U_\perp$ leads to different effects for the spin-polarized and unpolarized Luttinger liquids. First, with spin polarization the momentum and energy conservation suppresses scattering processes at the Fermi level. In other words, the cos term of the *sine-Gordon* equation is suppressed at low T. Second, the unpolarized Luttinger liquid has charge-spin separation, i.e., the 4×4 matrix [Eq. (21)] can be reduced to two 2×2 matrices, one corresponding to the charge degrees of freedom and the other to the spin degrees of freedom, for all values of V and $U_\parallel$. A direct comparison of the spin-polarized and unpolarized situations is then difficult.

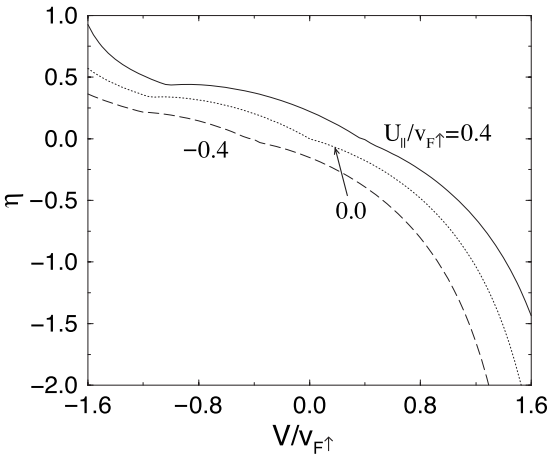

FIG. 4. Leading critical exponent for the transresistivity η as a function of the forward-scattering amplitude V for $U_\parallel=0.4v_{F\uparrow}$ (solid curves), $U_\parallel=0.0$ (dotted curves), and $U_\parallel=-0.4v_{F\uparrow}$ (long-dashed curves). Here the bare Fermi velocities are $v_{F\uparrow}=1.0$ and $v_{F\downarrow}=0.5$, respectively.

For the unpolarized electron gas the ground-state phase diagram involves several possibilities of long-range order, ranging from singlet and triplet superconductivity to spin- and charge-density waves with the formation of a spin gap. These ordered phases are less favorable in the spin-polarized system, e.g., in singlet superconductivity a pairing between carriers reversed spin and momentum is no longer possible (a Fulde-Ferrell-Larkin-Ovchinnikov-type state could be realized), the conditions for a spin gap are no longer favorable, and charge- and spin-density waves have to adapt themselves to two different Fermi momenta. Under these circumstances it is expected that a perturbative summation of bubble diagrams (as used here) is valid.

ACKNOWLEDGMENT

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APPENDIX: CONDUCTIVITY IN SECOND-ORDER PERTURBATION IN THE COULOMB INTERACTION

In the absence of phonons and impurities, the total current operator $j=j_\uparrow+j_\downarrow$ is a conserved quantity for Hamiltonian (6), i.e., $[j,H]=0$ or $\langle j\rangle=0$ if there are no external potentials. Consequently, the force operator acting on the up spins is opposite to that acting on the down spins, i.e., $d\langle j_\uparrow\rangle/dt=-d\langle j_\downarrow\rangle/dt$ or $A_\uparrow=-A_\downarrow$ with $A_s=[j_s,H]$. The matrix memory function can then be expressed in terms of a single scalar function $\chi(z)$,

$$\hat{\chi}(z)=\chi(z)\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}. \tag{A1}$$

Defining $\hat{N}(z)=\hat{\chi}^0\hat{M}(z)$ we obtain

$$\hat{N}(z)=\frac{z\chi(z)}{1-\chi(z)m\left(\frac{1}{n_\uparrow}+\frac{1}{n_\downarrow}\right)}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}. \tag{A2}$$

Below we calculate $\chi(z)$ to second order in the Coulomb interactions.

1. Forward scattering

At this point we only consider forward scattering, i.e., in H' we set $U_\parallel=U_\perp=0$ and keep only the term with V. In the absence of impurities the force operator acting on the spin component s is then

$$A_s=[j_s,H_V]=\frac{V}{mL}\sum_{kk'q}q(a^\dagger_{k+qs}a_{ks}b^\dagger_{k'-q\bar{s}}b_{k'\bar{s}}-a^\dagger_{k+q\bar{s}}a_{k\bar{s}}b^\dagger_{k'-qs}b_{k's}). \tag{A3}$$

The imaginary (dissipative) part of the equal spin current-current correlation function is given by $\omega^2\langle\langle j_s;j_s\rangle\rangle''_\omega=-\langle\langle A_s;A_s\rangle\rangle''_\omega$,

$$\omega^2\langle\langle j_s;j_s\rangle\rangle''_\omega=-\left(\frac{V}{mL}\right)^2\int\frac{d\omega_1}{\pi}\int\frac{d\omega_2}{\pi}\times 2\pi\delta(\omega-\omega_1-\omega_2)\left[\coth\left(\frac{\omega_1}{2T}\right)+\coth\left(\frac{\omega_2}{2T}\right)\right]\times\sum_{kq}q^2\langle\langle a^\dagger_{k+qs}a_{ks};a^\dagger_{ks}a_{k+qs}\rangle\rangle''_{\omega_1}\times\sum_{k'}\langle\langle b^\dagger_{k'-q\bar{s}}b_{k'\bar{s}};b^\dagger_{k'\bar{s}}b_{k'-q\bar{s}}\rangle\rangle''_{\omega_2}+(s\leftrightarrow\bar{s}). \tag{A4}$$

The two correlation functions in Eq. (A4) are now evaluated for the noninteracting system, yielding

$$\langle\langle j_s;j_s\rangle\rangle''_\omega=-2\left(\frac{V}{2\pi m}\right)^2L\frac{\omega^2}{(v_{Fs}+v_{F\bar{s}})^5}\times\left[\coth\left(\frac{v_{Fs}}{v_{Fs}+v_{F\bar{s}}}\frac{\omega}{2T}\right)+\coth\left(\frac{v_{F\bar{s}}}{v_{Fs}+v_{F\bar{s}}}\frac{\omega}{2T}\right)\right]. \tag{A5}$$

Consequently, as $T\to 0$, $M''(\omega)$ is proportional to $|\omega|^3$. According to the definition of relaxation time, Eq. (5), forward scattering alone does not lead to a relaxation process that is exponential with time.

The single-particle spectral function and the transverse structure factor in a ferromagnetic Luttinger liquid with forward scattering only have been studied by Bartosch *et al.*[31] They obtain singularities with nonuniversal exponents for momentum transfer $k_{F\uparrow}-k_{F\downarrow}$. These singularities are not seen directly in the current-current correlation since the function has no explicit momentum dependence.

2. Backward scattering

Here we first consider large momentum-transfer scattering without spin flip, i.e., in H' we set $V=U_\perp=0$ and keep only the term with $U_\parallel$. In the absence of impurities the force operator acting on the spin component s is $A_s=[j_s,H_{U_\parallel}]=0$, i.e., the terms for the current of forward moving electrons cancels the one of backward moving particles. Hence, the $U_\parallel$ interaction alone does not contribute to the resistivity. This is because the interaction conserves the momentum and does involve only one spin component.

The $U_\perp$ interaction, on the other hand, gives rise to an exponentially activated process with temperature (Arrhenius law) since the simultaneous transfer of up-spin and down-spin electrons from one Fermi point to the other requires a minimal energy of $2(v_{F\uparrow}k_{F\uparrow}-v_{F\downarrow}k_{F\downarrow})$. We set $V=U_\parallel=0$ and keep only the term with $U_\perp$ in H'. Without impurities the force operator acting on the current of spin component s is then

$$A_s=[j_s,H_{U_\perp}]=\frac{U_\perp}{mL}\sum_{kk'qs'}ss'(k+q-k')\times a^\dagger_{k+qs'}b^\dagger_{k'-qs'}a_{k\bar{s}'}b_{k's'} \tag{A6}$$

and the imaginary part of the equal spin current-current correlation function is

$$\omega^2\langle\langle j_s;j_s\rangle\rangle''_\omega = -\langle\langle A_s;A_s\rangle\rangle''_\omega$$

$$= -2\int \frac{d\omega_1}{\pi}\int \frac{d\omega_2}{\pi} 2\pi\delta(\omega-\omega_1-\omega_2)$$

$$\times\left[\coth\left(\frac{\omega_1}{2T}\right)+\coth\left(\frac{\omega_2}{2T}\right)\right]\left(\frac{U_\perp}{mL}\right)^2$$

$$\times\sum_{kk'q}\langle\langle a^\dagger_{k+qs'}a_{k\bar{s}'};a^\dagger_{k\bar{s}'}a_{k+qs'}\rangle\rangle''_{\omega_1}(k+q-k')^2$$

$$\times\langle\langle b^\dagger_{k'-q\bar{s}'}b_{k's'};b^\dagger_{k's'}b_{k'-q\bar{s}'}\rangle\rangle''_{\omega_2}. \quad \text{(A7)}$$

The two correlation functions in Eq. (A7) contain operators with opposite spin in contrast to those in Eq. (A4). If again evaluated for the noninteracting system they yield the exponential activation for $k_{F\uparrow}\neq k_{F\downarrow}$ discussed above.

Hence, at $T=0$ a relaxation time cannot be defined in this case either [see Eq. (5)]. This is the consequence of the momentum conservation of the interactions. It is also worth noticing that to second order in perturbation there are no terms mixing the interaction terms. In conclusion, a finite relaxation time can only be obtained if the translational invariance of the electron system is broken, e.g., either by phonons or impurity scattering.

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